ENDOMORPHISM RINGS WITH TWO (OR FINITELY MANY) MAXIMAL IDEALS

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I will present the main ideas that appear in two papers [5, 6] written with Pavel Příhoda (Charles University in Prague). Our research is based on the observation that a number of classes $\mathcal{C}$ of indecomposable right modules have the following two properties: (1) the endomorphism ring of every module in $\mathcal{C}$ has at most two maximal right ideals; (and consequently) (2) the *weak Krull-Schmidt theorem* holds for $\mathcal{C}$, that is, there exist two equivalences $\sim$ and $\equiv$ on the class $\mathcal{C}$ for which the following statement is true: "Let $U_1, \dots, U_n, V_1, \dots, V_m$ be modules in $\mathcal{C}$. Then $U_1 \oplus \cdots \oplus U_n \cong V_1 \oplus \cdots \oplus V_m$ if and only if $m = n$ and there are two permutations $\sigma, \tau \in S_n$ such that $U_i \sim V_{\sigma(i)}$ and $U_i \equiv V_{\tau(i)}$ for every $i = 1, \dots, n$". For instance this holds for the class $\mathcal{C}$ of biuniform modules [2], i.e., the modules of Goldie dimension one and dual Goldie dimension one; it holds for the class $\mathcal{C}$ of all kernels of non-injective morphisms between indecomposable injective modules [4]; it holds for the class $\mathcal{C}$ of couniformly presented modules [1, 3]. We will present what happens for an arbitrary class $\mathcal{C}$ of indecomposable modules whose endomorphism rings have at most two maximal right ideals [6] (Is this sufficient for the weak Krull-Schmidt Theorem to hold? This would be a two-dimensional analogue of the Krull-Schmidt-Remark-Azumaya Theorem...), or more generally, for a class $\mathcal{C}$ of modules whose endomorphism rings have finitely many maximal right ideals, all of them two-sided ideals [5].

References

[1] B. Amini, A. Amini and A. Facchini, *Equivalence of diagonal matrices over local rings*, J. Algebra **320** (2008), 1288-1310.

[2] A. Facchini, *Krull-Schmidt fails for serial modules,* Trans. Amer. Math. Soc. **348** (1996), 4561–4575.

[3] A. Facchini and N. Girardi, *Couniformly presented modules and dualities*, to appear in the Proceedings of the "Conference on Algebra and its Applications" in honor of 70th Birthday of S. K. Jain, Dinh Van Huynh and Sergio R. López Permouth Eds., Birkhäuser Verlag, Basel, Heidelberg, London, New York, 2009.

[4] A. Facchini, M. T. Koşan and T. Özdin, *Kernels of morphisms between indecomposable injective modules*, to appear, 2008.

[5] A. Facchini and P. Příhoda, *Endomorphism rings with finitely many maximal right ideals*, to appear, 2009.

[6] A. Facchini and P. Příhoda, *Endomorphism rings with at most two maximal right ideals*, to appear, 2009.

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