THE ϕ_J POLAR DECOMPOSITION

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Let $J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \in M_{2n}(\mathbb{C})$ and define the function

$$\phi_J : M_{2n}(\mathbb{C}) \to M_{2n}(\mathbb{C}) \text{ by } \phi_J(A) = J^{-1}A^TJ.$$

If $\phi_J(A) = A$, then A is said to be ϕ_J *symmetric* or *skew-Hamiltonian.* If $\phi_J(A) = A^{-1}$, then A is said to be ϕ_J *orthogonal* or *symplectic.* A matrix $A \in M_{2n}(\mathbb{C})$ is said to have a ϕ_J *polar decomposition* if there exist ϕ_J symmetric X and ϕ_J orthogonal Y such that $A = XY$.

It is known that every nonsingular $A \in M_{2n}(\mathbb{C})$ has a ϕ_J polar decomposition and that a matrix with a ϕ_J polar decomposition is necessarily of even rank. In joint work with D.I. Merino and D.C. Pelejo, we provide necessary and sufficient conditions for a matrix of even rank at most 4 to have a ϕ_J polar decomposition and reduce the study of the general case (rank $2k$) to matrices with a particular form.

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