

Cellular covers of cotorsion-free modules

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Abstract

Recall that a homomorphism of R -modules $\pi : G \rightarrow H$ is called a *cellular cover* over H if π induces an isomorphism

$$\pi_* : \text{Hom}_R(G, G) \cong \text{Hom}_R(G, H),$$

where $\pi_*(\varphi) = \pi\varphi$ for each $\varphi \in \text{Hom}_R(G, G)$ (where maps are acting on the left). First we show that every cotorsion-free R -module of rank $\kappa < 2^{\aleph_0}$ is realizable as the kernel of some cellular cover of rank $3\kappa + 1$ (or 3, if $\kappa = 1$). The proof is based on Corner's classical idea of how to construct torsion-free abelian groups with prescribed countable endomorphism rings. This complements results by Buckner–Dugas [1]. On the other hand, we are able to prescribe every cotorsion-free R -module as the cokernel of arbitrarily large cellular covers, if some rigid conditions are satisfied. This improves results by Fuchs–Göbel [4] and Farjoun–Göbel–Segev–Shelah [3].

References

- [1] J. Buckner and M. Dugas, *Co-local subgroups of abelian groups*, pp. 29–37 in *Abelian groups, rings, modules and homological algebra*; Proceedings in honor of Enochs, Lect. Notes Pure Appl. Math. **249**, Chapman & Hill, Boca Raton, FL 2006.
- [2] E. Dror Farjoun, R. Göbel, and Y. Segev, *Cellular covers of groups*, *J. Pure Appl. Algebra* **208** (2007), no. 1, 61-76.
- [3] E. Dror Farjoun, R. Göbel, Y. Segev, and S. Shelah, *On kernels of cellular covers*, *Groups Geom. Dyn.* **1** (2007), no. 4, 409-419.
- [4] L. Fuchs and R. Göbel, *Cellular covers of abelian groups*, *Results in Mathematics* (2008).

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