FLAT MITTAG-LEFFLER MODULES AND DRINFELD VECTOR BUNDLES

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Mittag-Leffler modules were introduced by Raynaud and Gruson already in 1971 [8], but only recently, Drinfeld suggested to employ them in infinite dimensional algebraic geometry. While the class of (infinitely generated) projective modules can be used to define (infinite dimensional) vector bundles, the class of flat modules is too big to admit a reasonable theory for the corresponding notion of a flat quasi-coherent sheaf on a scheme. Drinfeld's idea [3] is that the class $\mathcal{D}$ of flat Mittag-Leffler modules should yield a more general, but still tractable, subclass of the class of flat quasi-coherent sheaves. Two questions naturally arise here:

(i) What is the structure of flat Mittag-Leffler modules over particular (notably commutative noetherian) rings R?

(ii) Can we employ the class $\mathcal{D}$ in developing homological algebra and homotopy theory (via Quillen model category structures etc.) for complexes of quasi-coherent sheaves on a scheme?

In [7] we have recently answered both these questions by proving for any ring that flat Mittag-Leffler modules coincide with the $\aleph_1$-projective ones in the sense of [4], and by showing, for any non-perfect ring, that the class $\mathcal{D}$ is not deconstructible. The latter implies that the answer to (ii) is negative. Moreover, in the particular case of $R = \mathbb{Z}$, the results of [5] and [6] can be adapted to give a stronger negative result: The class of all $\aleph_1$-free groups is not precovering, [2].

References

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